Abstract—Deploying Autonomous Underwater Vehicles (AUVs) is a necessity to enable a range of civilian/military underwater applications; yet, achieving a reliable coordination among the vehicles is a challenging issue due to the time- and space-varying characteristics of the acoustic communication channel. The design of a Medium Access Control (MAC) based on a probabilistic Space Division Multiple Access (SDMA) method for short/medium distances (less than 2 km) is presented. This method considers the inherent vehicle position uncertainty due to the inaccuracies in models and the drift of the vehicles. It minimizes the acoustic interference statistically by considering the angular position of neighboring vehicles via a two-step estimation and by keeping the transmitter antenna’s beamwidth of each vehicle at an optimal value. Such value is chosen considering three contrasting goals, i.e.: (i) spreading the signal beam towards the vehicle to combat position uncertainty using a coarse estimation; (ii) focusing the beam to reduce acoustic energy dispersion through a fine estimation; and (iii) minimizing interference to other vehicles. Simulation results in a sparse underwater network show that this approach mitigates interference, reduces the probability of retransmission, and achieves higher data rates over conventional underwater MAC techniques.

Index Terms—Underwater Acoustic Communications; Space Division Multiple Access; Position Uncertainty; Autonomous Vehicles.

1 Introduction

Overview: Underwater wireless acoustic networks, which are composed of static sensors and mobile vehicles, underpin the underwater world and are instrumental to support next-generation ocean-observation systems, to enable both civilian and military applications, and to pave the way towards the futuristic Underwater Internet of Things (UWIoT) paradigm. Oceanographic data collection, ocean pollution monitoring, offshore exploration, tsunami detection/disaster prevention, assisted navigation, and tactical surveillance are examples of some of such applications [2], [3]; for most if not all of these applications achieving communication and coordination is key, which in turn calls for the ability to transmit reliably signals underwater. This problem is still challenging in the harsh underwater environment in which Radio-Frequency (RF) waves are absorbed for distances above a few tens of meters, optical waves require narrow laser beams and suffer from scattering as well as ocean wave motion, communication through magnetic induction is only feasible up to a few meters, and acoustic waves lead to a communication channel that is very dynamic, prone to fading, spectrum limited with passband bandwidths of only a few tens of kHz due to high transmission loss at frequencies above 50 kHz, and affected by non-Gaussian noise [4]. In addition, while the acoustic waves can propagate underwater up to several tens of kilometers—making the communication technologies based on them the only possible for distances above a hundred meters—they are known to be affected by a variety of factors including temperature, salinity, and pressure of the body of water traversed.

Motivation: One of the main challenges for the effective coordination of a team of autonomous vehicles underwater is how to design a fair and efficient Medium Access Control (MAC) protocol tailored to the harsh underwater acoustic environment. Due to the unique characteristics of the propagation of acoustic waves in the water, in fact, existing terrestrial MAC solutions are unsuitable for this environment [2]. One promising yet unexplored MAC technique for sparse underwater networks is Space Division Multiple Access (SDMA) which exploits the signal beam directivity and the spatial separation of the vehicles. It makes use of the fact that vehicles/mobile users can be served simultaneously when they are not located in the same area, so that the radiated energy for each user can be separated in space [5]. Among the possible channelized MAC techniques, SDMA is more efficient when the network is sparse—as in the underwater environment—than Frequency Division Multiple Access (FDMA), which has limited acoustic bandwidth per channel, Time Division Multiple Access (TDMA), which requires long time guards and high signaling overhead (especially when some of the users are mobile and/or the body of water is large [6]), and Code Division Multiple Access (CDMA), which is affected by a low data rate [7]–[9].

Notwithstanding these considerations, there are strong arguments against a brute-force application of terrestrial SDMA to the underwater acoustic environment. In such MAC scheme, in fact, the expected increase in the data rate is highly affected by the accuracy of the Channel State Information (CSI) at the transmitter; therefore, a considerable amount of effort should be dedicated to compensate for the partial information at the transmitter in a (possibly) rapidly changing underwater channel. Furthermore, a dramatic reduction in the throughput is observed if the feedback is delayed, since the CSI becomes outdated. Noticeably, in underwater acoustic channels, CSI is usually unknown to both transmitter and receiver. Moreover, in the terrestrial SDMA, every

1. The network sparsity assumption here—leading to a high likelihood of spatial separation between vehicles—comes from the observation that the vehicle density in a body of water is often low due to cost and scalability.
user can provide its own real-time position information, which is not feasible underwater as Global Positioning System (GPS) does not work and the position information is not readily available. Even if we had the users’ locations, applying them would still not be feasible because achieving perfect pointing between transmitter and receiver is challenging due to the position uncertainty of the vehicles and the nature of acoustic wave propagation. Furthermore, considering the effect of ocean currents on the vehicle, inaccuracies in position estimation increases the position uncertainty [1], [10]; and this uncertainty becomes worse over time when the vehicle stays longer underwater due to error propagation, which leads to non-negligible drifts in the vehicle’s position and thus making conventional SDMA inapplicable.

Related Work: During the past few years, several MAC protocols have been proposed for underwater communications. Time sharing-based solutions are exploited in many real underwater scenarios. However, they will not be very efficient if the long propagation delay of the channel is not considered [11], [12]. Authors in [13] proposed a distributed and energy-efficient MAC protocol called Tone Lohi (T-Lohi), as an energy efficient tone-based contention algorithm. This technique shows low channel utilization, when the number of nodes increases. A delay tolerant MAC protocol (DTMAC) was proposed in [14], in which the solution applies short-packets traffic to combat the effect of long propagation delay and mobility in sparse networks. A collision-free TDMA scheduling was discussed in [15] to improve the throughput by considering large propagation delays. Recently, the collected data at sea experiments confirms that there is no unique MAC solution for all the scenarios and configurations under various conditions [16]. Other random- and controlled-access MAC protocols such as Carrier-sense Multiple Access (CSMA) transmit multiple packets through the same underwater channel, which might lead to packet collisions at the receiver. The other method, which has both the carrier sensing and collision avoidance mechanisms, is Floor Acquisition Multiple Access (FAMA). The objective of this protocol is to ensure that a single sender reserves the channel via an RTS (Request to Send)/CTS (Clear to Send) handshake before transmitting a packet.

In this section, we briefly review conventional space sharing MAC algorithms and their related challenges, while keeping in mind that they cannot be directly used in underwater channels.

Assuming spatial separation of the users, sectorized antenna can be a primitive application of terrestrial SDMA [5]. SDMA-based Smart antennae in mobile networks can improve the network capacity. A robust and self-organizing terrestrial SDMA is proposed for mobile ad-hoc networks in [17], where it is shown that the network bandwidth efficiency depends on the number of mobile users. In [18], an opportunistic terrestrial-based SDMA with threshold feedback algorithm for enforcing the sum feedback rate constraint was proposed. All these approaches still need the channel information to be fed back to the transmitter, which is not always feasible underwater due to the long propagation delays. Authors in [19] proposed a zero-forcing precoding with partial CSI, which is known at the transmitter in the terrestrial ad-hoc networks. In [20], beamforming was proposed for multiuser multi-antenna SDMA downlink systems. In [21], the authors presented a hybrid architecture for downlink beamforming with phased antenna arrays in indoor SDMA channels as a new generation of broadband terrestrial personal and local area networks. Recently, multi-beam smart antenna array system based on SDMA was presented, which is a promising candidate for next generation wireless communications as it enables the antennas to steer the energy towards a desired direction [22].

To implement any spatially-divided MAC protocol for the underwater channel, as we mentioned earlier, one of the main challenges to face is the inaccuracy and uncertainty in localization models of the underwater vehicles. Short Baseline (SBL) is one of the most common ways to localize vehicles underwater, in which the position estimate is performed via external transponder arrays. Long Baseline (LBL) system, similarly to SBL, also uses tethered external transponder arrays with fixed locations [23] in farther distances. Dead-reckoning estimation of position is based on accumulated measurement of the velocity compared to the surface. AUV Aided Localization (AAL) methods were also introduced in the literature, in which the distances to the AUV is estimated by each node, while the AUV is at different locations [24], [25]. In [10], an approach has been proposed to predict vehicles’ position through statistical method. This method also estimates the position uncertainty of the vehicles and designs a routing protocol based on the vehicles’ confidence region. Given the randomness of underwater channel and its long propagation delay, interference distribution under various signal propagation models was discussed in [12]. However, our proposed solution aims at mitigating the interference between the users caused by the position uncertainty via spatial and time division methods.

Our Contributions: Any channel access method that exploits a deterministic approach for interference mitigation/cancellation would not be an efficient solution underwater as it ignores the inherent position uncertainty of the vehicles caused by drifts, model errors, and unbounded errors, thus leading to performance degradation. For these reasons, we present a novel probabilistic and spatially-divided MAC to cancel/ alleviate the interference while the inherent position uncertainty of vehicles is considered in a sparse underwater mobile network. An Angle of Departure (AOD)-based solution forms separate spatial beams via a probabilistic approach towards the target vehicles. Since the vehicles are mobile, a two-stage estimation scheme is required to calculate the beam parameters, i.e.,: (i) a coarse interval estimation and (ii) a fine estimation via unscented Kalman filtering to update the beam parameters for each antenna. An optimization problem mitigates the statistical interference between the expected overlapped vehicles by keeping the transmit beamwidth and direction within a desirable range. In the case the vehicles are entirely overlapped in space, we propose a hybrid probabilistic time and space MAC scheme, called T-SDMA, which takes time into account besides space, and outperforms conventional TDMA methods in terms of rate efficiency.

Article Outline: The remainder of this article is organized as follows. In Sect. 2, we provide a discussion on the assumptions made in this work. In Sect. 3, we introduce the proposed system and provide solutions for probabilistic SDMA, where both spatially separable and non-separable scenarios are considered. In Sect. 4, we present our simulation results and discuss the benefits of our solution. Finally, in Sect. 5, we draw the main conclusions.

2 Underwater Acoustics

While sound waves travel through the underwater medium, part of the acoustic/elastic energy is absorbed; a well-known expression that models the medium absorption coefficient as a function of frequency is: \( a(f) = (0.11 f^2)/(1 + f^2) + (44 f^2)/((4100 + f^2) + 2.75 \times 10^{-4} f^2 + 0.003) \) [26]. In this
emphatic formula, $10 \log_{10} a(f)$ gives the channel attenuation in dB/km. Propagation loss can be modeled via $P_a = \frac{\pi f D \sigma_a}{1 + \frac{2f_D}{f}}$, in which $\zeta$, $D$, and $\sigma_a$ stand for the scattering loss, distance, and spreading loss, respectively [27].

Let us assume that the transmission occurs at short/medium ranges and that the direct beams are dominant over the reflected beams from the surface and the bottom of the sea so that the receiver is not severely affected by multipath. For farther distances (above a few kilometers) and based on the sound-speed profile, the acoustic rays bend towards the region of lower acoustic speed (the so-called “laziness law”). This effect changes the Angles Of Departure/Arrival (AOD/AOA) and their estimations. Using the Bellhop model [28] and considering a typical deep-water case, Fig. 1 illustrates the sound-speed profile (left) and the acoustic ray tracing (right) in the underwater channel for a sample source at a depth of $\sim 0.9$ km and a water temperature of 39°C. The bending effect can be observed at distances of a few kilometers; however, staying within a short/medium range (less than 5 km), such bending is not notable, which explains the philosophy behind our signaling method in which the vehicle is steered via beamforming.

To consider the time variability of an underwater acoustic channel, assume it varies after approximately $t_e$ seconds (channel coherence time). This parameter can be defined by Clarkes model [5] as $t_e = \sqrt{\frac{9}{16\pi^2 f^2}} \approx 0.423/f_d = 0.423/(\alpha_d f_c)$, where $f_d$ is the Doppler shift, $f_c$ is the carrier frequency, and $\alpha_d$ represents the Doppler scaling factor. Let $D$ be the distance between the buoy and the vehicle, then the round trip delay time $t_D$ for the distance $2D$ should be less than $t_e$. It is easily shown that for an underwater channel with a specific sound profile and with $f_c = 20$ kHz, $\alpha_d = 3 \times 10^{-5}$, and for distances greater than $D \approx 500$ m, the time for receiving feedback CSI signal could be larger than the coherence time of the channel [29]. It is worth mentioning that the proposed technique in Sect. 3 is not channel dependent, so the information sent back to the transmitter is usable for a longer time compared to channel variations. Furthermore, as discussed in [10], by assuming that ocean currents are unknown, the vehicle’s drifting in the horizontal plane is identically and independently distributed (i.i.d.) and follows a normal distribution, which makes the horizontal projection of its confidence a circular region. Regarding the vehicle’s movement along its trajectory, the uncertainty region is concluded to be a cylinder [10].

There are multiple methods for underwater vehicles to be localized that can be generally categorized as inertial dead reckoning, acoustic transponders and modems, and geophysical meth-

![Fig. 1. Bellhop ray tracing (right box) for a standard sound-speed profile (left box) indicating how acoustic beams travel through the underwater acoustic channel [28]; notice how the beams are almost straight for a distance of a few kilometers; however, in a short/medium range (less than 5 km) when the transmitter is at a depth of 0.9 km.](image1)

![Fig. 2. System framework depicting the interaction between different parts, so-called macro-states, of the system.](image2)

ods [30]. The selection of the localization method is dependent on the application, the environment and the desired accuracy. In some regions, depth sensors can be implemented to provide information about vertical position of the vehicles. In some applications, the receiver antenna arrays can be utilized to estimate the AUV’s location. In [31], we utilized Acoustic Vector Sensors (AVS) to estimate the angle of arrival. In this paper, we follow [10] and choose dead reckoning as the initial localization technique, although the localization technique does not directly impact our proposed solution. Dead reckoning method is subject to cumulative errors, but as a classic location estimator and because of its simplicity of implementation, it is still a widely-used solution in AUVs [30]. Each vehicle estimates its trajectory and position, using its own location estimates and considering the inherent position uncertainty of objects underwater. Every $\Delta t$ seconds, each vehicle estimates its current location by measuring its velocity and using the previous estimated locations. Vehicles adopt a polling model, as will be explained in the algorithm, to send back the measured position samples through a feedback channel. Notice that the type of vehicles that can benefit from this research depends on the application, but it does not directly impact our proposed solution. Buoyancy-propelled gliders—which follow a sawtooth-shaped glide path—move not as fast as conventional AUVs (a fraction of a meter per second); however, they are extremely efficient in terms of power consumption making them suitable for background monitoring missions, whereas propeller-driven AUVs are capable of operating at higher speeds.

3 OUR SOLUTION

In this section, we present our solution and provide more details for different parts of the system. Fig. 2 shows the system framework through the interaction between its macro-states. We define the notion of macro-states to distinguish them from the definition of states, which will be used later for beam’s parameters. First, we form separate spatial beams probabilistically towards the target based on coarse and fine estimations, if we do not expect any severe interference; otherwise, the system transits to the next macro-state, which consists in the statistical interference cancellation. If the probability of interference/miss trade-off is satisfactory, then we shift to the transmit macro-state. In the case that the vehicles are non-spatially separable, a hybrid probabilistic time and space MAC scheme, called T-SDMA, will be performed. The detail of each macro-state will be discussed as follows. First, in Sect. 3.1, we introduce the initialization procedure and the coarse estimation for our probabilistic SDMA via interval estimation of the antenna’s parameters. Then, in Sect. 3.2, we present
TABLE 1
Notations and Mathematical Terms- Coarse Estimation

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a(f)$</td>
<td>Medium absorption coefficient</td>
</tr>
<tr>
<td>$P_o$</td>
<td>Propagation loss at distance $D$</td>
</tr>
<tr>
<td>$\varsigma$, $\varpi$</td>
<td>Scattering and spreading loss parameters</td>
</tr>
<tr>
<td>$t_D$</td>
<td>Round trip delay</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Coherence time</td>
</tr>
<tr>
<td>$f_d$, $\alpha_d$</td>
<td>Doppler shift and scaling factor</td>
</tr>
<tr>
<td>$n$, $N$</td>
<td>Location sample index, total location samples</td>
</tr>
<tr>
<td>$\mathcal{J}$</td>
<td>Total number of vehicles</td>
</tr>
<tr>
<td>$\Delta z$, $\Delta x$</td>
<td>Vertical and horizontal location drift</td>
</tr>
<tr>
<td>$(i)$, $(j)$</td>
<td>Neighboring vehicles</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Vector of parameters</td>
</tr>
<tr>
<td>$T_x$</td>
<td>Buoy’s antennae</td>
</tr>
<tr>
<td>$m$</td>
<td>Antenna index</td>
</tr>
<tr>
<td>$\theta^{(i)}$, $\phi^{(i)}$</td>
<td>Angle in elevation plane, angle in azimuth plane</td>
</tr>
<tr>
<td>$h_L$, $h_U$, $R$</td>
<td>Cylinder’s lower and upper height, and radius</td>
</tr>
<tr>
<td>$\theta^{(i)}_L$, $\theta^{(i)}_U$</td>
<td>Lower and upper boundaries of vehicle $(j)$</td>
</tr>
<tr>
<td>$W^{(j)}_b$</td>
<td>HPBW towards vehicle $(j)$</td>
</tr>
<tr>
<td>$\Gamma^{(j)}_g$</td>
<td>AOD towards vehicle $(j)$</td>
</tr>
</tbody>
</table>

Table 1 summarizes the main mathematical terms used in the coarse estimation section.

3.1 Coarse Confidence Interval Estimation

In this step, we explore how the estimated position of the vehicles are used to achieve a coarse estimation of antenna parameters in a centralized fashion. Fig. 3 shows the general configuration of the system including buoy’s antenna arrays, vehicles, and their cylindrical position uncertainty regions. A second set of arrays, which is not depicted in this figure, is implemented on the other side of buoy to cover the spherical space around it. Assume the neighboring vehicles $i$, $j$, and $k$ follow different trajectories. To calculate the uncertainty region, the vehicle $j$ broadcasts its $N$th location sample at time $t$ as $\text{loc}_{m}(t) = [x^{(j)}_n, y^{(j)}_n, z^{(j)}_n]_{n=1}^N$. Assume the initial AODs towards two separate vehicles $i$ and $j$ are identified as $\theta^{(i)}$ and $\theta^{(j)}$. Since the variation of a vehicle’s position inside the uncertainty region can be defined as a random variable [10], it is inferred that the variation of the angles at the buoy's transmitter is also a random variable with unknown mean and variance. We claim that each angle is a result of a drift in the $n$th location sample as $\theta^{(j)}_{\alpha}(t) = \tan^{-1}(z^{(j)}_n/(t - t_{D}/2))$ where $t_{D}/2$ shows the delay in transmission, as discussed in Sect. 2. For simplicity of notation, let us drop the superscript $j$ in $z^{(j)}_n(t - t_{D}/2) = \Delta z$ and $x^{(j)}_n(t - t_{D}/2) = \Delta x$, where $\Delta z$ and $\Delta x$ stand for the vertical and horizontal drifts in the vehicle’s location after $t_{D}/2$. For example, for a distance of $D = 1000$ m, the delay equals $t_{D}/2 \approx 0.66$ s and the vehicle moves $\Delta x \approx 0.16$ m, which is negligible compared to the transmission distance considering a vehicle’s constant speed of $0.25$–$0.5$ m/s [10]. Based on the Taylor's polynomial approximation, $\tan^{-1}(z^{(j)}_n/x^{(j)}_n) \approx z^{(j)}_n/x^{(j)}_n - 1/3(z^{(j)}_n/x^{(j)}_n)^3 + 1/5(z^{(j)}_n/x^{(j)}_n)^5 - ...$, where $z^{(j)}_n$ is the vehicle’s vertical drift due to its position uncertainty. This value is very small in comparison with $x^{(j)}_n$, which is the horizontal distance between the vehicle and the buoy; consequently, $\theta^{(j)}_{\alpha} \approx z^{(j)}_n/x^{(j)}_n$. Besides, [32] provides approximations to demonstrate in practice that many of the ratios of normal random variables are normally distributed if the denominator of $z/x$ is positive and its coefficient of variation is very small. Based on the numerical calculations in [33], we can conclude that $\theta^{(j)}_{\alpha}$ follows a normal distribution.

Definition 1. Let $X = (X_1, \ldots, X_n)$ be a random sample observation from a distribution with parameter $\Theta$, then a random variable $\mathcal{V}(X_1, \ldots, X_n, \Theta)$ is called a pivotal quantity if its distribution is independent of all parameters $\Theta$ [34].

Definition 2. An interval estimate of a parameter $\Theta$, for any random sample observation $X$, is defined by the pair of $L_{\Theta}(X)$ and $U_{\Theta}(X)$, where $L_{\Theta}(X) \leq \Theta \leq U_{\Theta}(X)$. This interval, together with the probability $\Pr(\Theta \in [L_{\Theta}(X), U_{\Theta}(X)])$, is called confidence interval [34].

Statement 1. Since $\theta^{(j)}_{\alpha}$’s are random samples of a normal distribution with $\mathcal{N}(\mu_{\theta}, \sigma_{\theta}^2)$, from the statistical inference theorem [34] the mean $\bar{\theta}$ and the standard deviation of samples $S_\theta$ are also independent random variables and $\bar{\theta}$ has a normal distribution,
of interest [36]. A configurable software-defined platform, which utilizes a broadband phased array transducer, can achieve the required goals in a single unit on the fly, while the cost of having such multiple functions reduces [37].

### 3.2 Fine Estimation

As the vehicle moves, new measurements are acquired periodically over time and a fine estimation is required on top of the coarse estimation to reduce the energy dispersion and to support vehicle’s mobility as explained below. We propose a two-stage solution to handle the estimation in a continuous manner when the vehicles move and take new location samples. The calculated parameters and predictions by interval estimation should be updated by a Kalman Filtering in order to track the vehicles. The following statements and theorems study the cases in details.

#### Statement 3. Let \( \Omega \) be the space of infinite and countable states; the stochastic process \( \{ \gamma_n \}_{n \in N} \), whose components are in \( \Omega \), is said to possess the Markov property if the probability follows

\[
\Pr[\gamma_{n+1} = C_{n+1} | \{ \gamma_0 = C_0, ..., \gamma_n = C_n \}] = \Pr[\gamma_{n+1} = C_{n+1} | \gamma_n = C_n].
\]

**Theorem 1.** Given every \( \{n\}_{n \in N} \) sample observations, samples’ mean \( \tilde{\theta}_n \) and standard deviation \( S_{\theta,n} \) are sequence of random variables and possess the Markov Property when the new angle sample is considered and the samples are accumulated.

**Proof.** While the accumulation is performed, we have \( N \) samples at state \( k \) and \( N+1 \) samples at state \( k+1 \). For these states we can write \( \tilde{\theta}_k = \frac{1}{N} \sum_{n=1}^{N} \theta_n \) and \( \tilde{\theta}_{k+1} = \frac{1}{N+1} \sum_{n=1}^{N+1} \theta_n \). Therefore, \( \Pr[\tilde{\theta}_{k+1} | \{ \theta_0, ..., \theta_{k+1} \} ] = \Pr[\tilde{\theta}_{k+1} | \theta_k] \). Similarly, for the estimated standard deviation we can conclude that, \( S_{\theta,k+1} = \left[ \frac{1}{N+1} \sum_{n=1}^{N+1} (\theta_n - \tilde{\theta}_{k+1})^2 \right]^{1/2} = \left[ \frac{1}{N+1} \sum_{n=1}^{N} \left( \theta_n - N\tilde{\theta}_{k+N+1}/N \right) + \left( \theta_{N+1} - \tilde{\theta}_{k+N+1} \right)^2 \right]^{1/2} = \left[ \frac{1}{N+1} \sum_{n=1}^{N} \left( \theta_n - (N + 1)\tilde{\theta}_{k+N+1}/(N+1) \right)^2 \right]^{1/2} \). If \( N \) is large enough, then \( S_{\theta,k}^2 \) is replaced by \( N S_{\theta,k}^2 \), then

\[
S_{\theta,k+1} = \left[ \frac{1}{N+1} (N S_{\theta,k}^2 + N \left( \theta_{N+1} - \tilde{\theta}_k \right)^2 + (\theta_{N+1} - \tilde{\theta}_{k+1})^2) \right]^{1/2}.
\]

Since \( \sum_{n=1}^{N} \theta_n = N \tilde{\theta}_k \), the above equation results in

\[
S_{\theta,k+1} = \left[ \frac{1}{N+1} (N S_{\theta,k}^2 + N \left( \theta_{N+1} - \tilde{\theta}_k \right)^2 + (\theta_{N+1} - \tilde{\theta}_{k+1})^2) \right]^{1/2}.
\]

Now, since \( S_{\theta,k+1} \) is a function of two variables (i.e., \( \tilde{\theta} \) and \( S_{\theta} \)), where \( \tilde{\theta}_k \) and \( \theta_{N+1} \) are sequences of i.i.d. random variables, the Markov property holds by definition. Therefore, \( \Pr[\tilde{\theta}_{k+1} | \{ \tilde{\theta}_0, ..., \tilde{\theta}_{k-1}, \tilde{\theta}_k \} ] = \Pr[\tilde{\theta}_{k+1} | \tilde{\theta}_k] \). □

**Theorem 2.** Given every \( \{n\}_{n \in N} \) sample observations, samples’ mean \( \tilde{\theta}_n \) and standard deviation \( S_{\theta,n} \) are sequence of random variables and follow the Markov Property when the new angle sample slides the other samples as a fixed-size sliding window.
Proof. There are \( N \) samples at states \( k \) and \( k+1 \), if we use a fixed-size sliding window. Therefore, 
\[
\bar{\theta}_{k+1} = \frac{1}{N} \sum_{n=2}^{N+1} \theta_n = \frac{1}{N} \sum_{n=1}^{N} (\theta_n + \theta_{N+1} - \theta_1) = \bar{\theta}_k + \frac{1}{N} (\theta_{N+1} - \theta_1)
\]
and the Markov property holds. Similar calculations can be written for state \( k+1 \) of standard deviation as, 
\[
S_{\theta,k+1} = \left[ \frac{1}{N} \sum_{n=1}^{N} (\theta_n - \bar{\theta}_k - \frac{\theta_{N+1} - \theta_1}{N}) \right]^2 + \left( \frac{\theta_{N+1} - \theta_1}{N} \right)^2.
\]
By replacing \( \sum_{n=1}^{N} (\theta_n - \bar{\theta}_k)^2 \) by \( N S_{\theta,k}^2 \), we conclude that 
\[
S_{\theta,k+1} = \left[ \frac{1}{N} (NS_{\theta,k}^2 + (\theta_{N+1} - \bar{\theta}_k)^2 + (\theta_{N+1} - \bar{\theta}_k)^2 - \frac{2(\theta_{N+1} - \bar{\theta}_k)}{N} \sum_{n=1}^{N} (\theta_n - \bar{\theta}_k)) \right]^2.
\]
Since the last term in this equation equals zero, we have 
\[
S_{\theta,k+1} = \left[ \frac{1}{N} (NS_{\theta,k}^2 + (\theta_{N+1} - \bar{\theta}_k)^2 + (\theta_{N+1} - \bar{\theta}_k)^2 - \frac{2(\theta_{N+1} - \bar{\theta}_k)}{N} \sum_{n=1}^{N} (\theta_n - \bar{\theta}_k)) \right]^2.
\]
which proves that \( S_{\theta} \) also defines a Markov property.

Given the Markov property of beam parameters and the previous angle estimations for vehicle \( j \) at state \( k \), i.e., \( \hat{\Theta}_k^{(j)} \) and \( S_{\theta,k}^{(j)} \), the new estimations are updated at state \( k+1 \) as \( \bar{\Theta}_k^{(j)} \) and \( S_{\theta,k+1}^{(j)} \).

To stabilize the variations, we consider both the uncertainties in new samples and in the latest estimation of the previous state until the current state. We assign a weight to them by using Kalman Filtering. We update the current state of parameter vector \( \Theta \), which stands for the random variables \( [\Gamma_0(\hat{\theta}), W_0(S_\theta)] \) for vehicle \( j \) as follows,

\[
\begin{align*}
\Theta^{(j)}_{k+1} &= F(\Theta^{(j)}_k) + Z_k, \\
Y^{(j)}_{k+1} &= H(\Theta^{(j)}_{k+1}) + V_k,
\end{align*}
\]

where \( Z_k \) and \( V_k \) represent the process and measurement uncertainties, respectively. Functions \( F \) and \( H \) are the state-transition models, which are generally nonlinear. Based on the observed angle, the interval estimation calculates an estimation given \( C_{k+1}^{(j)} \)—which is a Markov state—with transition probability \( \Pr(\hat{\Theta}_k^{(j)} = C_{k+1}^{(j)}|\Theta^{(j)}_k = C_k^{(j)}) \). The probability of possible state for time instant \( k+1 \) equals to \( \Pr(\hat{\Theta}_k^{(j)} = C_{k+1}^{(j)}|\Theta^{(j)}_k = C_k^{(j)}) \cdot \Pr(\Theta^{(j)}_k = C_k^{(j)}) \), where \( \{\Theta^{(j)}_k\}_{k \in \mathcal{S}} \) takes its values from state space \( \mathcal{S} \). For vehicle \( j \), transition to the next state at time instant \( k+1 \) could be done via one of these three cases: Case I (A)—sample accumulation; Case II (S.W.)—fixed-size sliding window; or Case III (C.E.)—staying in the current estimate.

\[
\Pi^{(j)}_{k+1|k} = \begin{cases} 
\Pi^{(j)}_I, & \text{if } \{n\}_{k+1} = \{1, \ldots, N+1\}, \\
\Pi^{(j)}_Y, & \text{if } \{n\}_{k+1} = \{2, \ldots, N+1\}, \\
\Pi^{(j)}_{II}, & \text{if } \{n\}_{k+1} = \{1, \ldots, N\},
\end{cases}
\]

where \( \Pi^{(j)}_{k+1|k} \in [0, 1] \) is the transition probability for vehicle \( j \) and \( \sum_{i=1}^{III} \Pi^{(j)}_{k+1|k} = 1 \). Based on the following statement, two non-identical cases, Case II and Case III, are defined in (7).

**Statement 4.** Considering every observation \( \theta_{k+1} \) at time instant \( k+1 \), the behavior of the updated estimation \( Y^{(j)}_{k+1} \) in terms of mean and standard deviation is in general time dependent if the number of samples is not sufficiently large.

**Statement 5.** Let \( \theta_{k+1} \) be the observed angle sample at time instant \( k+1 \). Our MAC protocol decides on the next case as follows. Case I in (7) is superior when the updated estimations of the mean and the standard deviation need to be more accurate. Sample accumulation offers a better interval estimation for the beamwidth and AOD if the number of samples is bound to increase. This leads to a narrower beam, which possibly decreases the probability of interference but increases the probability of miss. On the other hand, keeping a constant number of samples, i.e., using a fixed-size sliding window, as in Case II, shows the time-dependent nature of the estimation as the vehicles change their positions. This fixed-size beamwidth does not increase the probability of miss. Case III is preferred to reduce the probability of miss, when the probability of interference is very small, i.e., when the vehicles are in space.

![Diagram](image-url) Fig. 4. Our MAC protocol decides on the next estimate of each vehicle when new samples are acquired. Coarse estimation is updated via one of the possible cases, as in Statement 5. The output is fed to the next block, which is an Unscented Kalman Filtering (UKF).
TABLE 2
Notations and Mathematical Terms- Fine Estimation Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_k$</td>
<td>Measurement noise</td>
</tr>
<tr>
<td>$Z_k$</td>
<td>Process noise</td>
</tr>
<tr>
<td>$Q_{Zk}, Q_V$</td>
<td>Covariance of noise $Z_k$ and $V_k$</td>
</tr>
<tr>
<td>$\mathcal{F}, \mathcal{H}$</td>
<td>State-transition models</td>
</tr>
<tr>
<td>$P_{i}^{(j)}(k)$</td>
<td>Transition probability for vehicle $(j)$</td>
</tr>
<tr>
<td>$y_{k}^{(j)}$</td>
<td>Observation estimation</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Mean of the parameter $\Theta$</td>
</tr>
<tr>
<td>$Q_{\theta_i}$</td>
<td>Covariance of the parameter $\Theta$</td>
</tr>
<tr>
<td>$K_G$</td>
<td>Kalman gain</td>
</tr>
<tr>
<td>$\zeta_{i,k}$</td>
<td>Sigma points for $(j)$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Number of the sigma points $\zeta$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Scaling parameter</td>
</tr>
<tr>
<td>$P_{i}^{(j)}$</td>
<td>Interference of $(i), (j)$</td>
</tr>
<tr>
<td>$\mu_i, \sigma_i$</td>
<td>Mean and standard deviation of interference</td>
</tr>
<tr>
<td>$\Theta'$</td>
<td>Vector of centered variables</td>
</tr>
<tr>
<td>$\Theta^*$</td>
<td>Optimum parameters</td>
</tr>
<tr>
<td>$W_{\theta_i}$</td>
<td>Initial HPBW</td>
</tr>
<tr>
<td>$\Gamma _{\theta}$</td>
<td>Initial AOD</td>
</tr>
<tr>
<td>$\theta_{i,L}, \theta_{i,U}$</td>
<td>Lower and upper boundaries</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Control parameter</td>
</tr>
<tr>
<td>$D$</td>
<td>Constraints domain</td>
</tr>
<tr>
<td>$(\theta^<em>)^</em>, (S^{<em>}, s^{</em>})^*$</td>
<td>Optimum mean and standard deviation</td>
</tr>
<tr>
<td>$R_{j}^{(j)}$</td>
<td>Retransmission rate (interference)</td>
</tr>
<tr>
<td>$R_{j}^{(j)}$</td>
<td>Retransmission rate (miss)</td>
</tr>
<tr>
<td>$R_{T,S}$</td>
<td>Time slot usage ratio for T-SDMA</td>
</tr>
<tr>
<td>$n_t, c \in \hat{c}$</td>
<td>Total number of time slots, clusters</td>
</tr>
<tr>
<td>$M_1$</td>
<td>Maximum number of vehicles in cluster</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Maximum number of clusters in inter-cluster group</td>
</tr>
<tr>
<td>$M_3$</td>
<td>Maximum vehicles per cluster in inter-clustered group</td>
</tr>
<tr>
<td>$\bar{T}$</td>
<td>Time slot usage ratio</td>
</tr>
<tr>
<td>$M_T$</td>
<td>Total number of time slots</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Rate efficiency</td>
</tr>
<tr>
<td>$\bar{R}_{T,S}$</td>
<td>Data rate per vehicle transmitting frame</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Time duration of a time slot</td>
</tr>
<tr>
<td>$N_p, N_l$</td>
<td>Number and length of packets</td>
</tr>
<tr>
<td>$SW$</td>
<td>Number of attempts (retransmissions)</td>
</tr>
</tbody>
</table>

Notation simplicity—as follows,

\[
\zeta_{i,k} = \begin{cases} 
\tilde{\Theta}_k + \beta \left( \sqrt[2]{\lambda \theta_{i,k}} \right), & i = 0, \\
\tilde{\Theta}_k - \beta \left( \sqrt[2]{\lambda \theta_{i,k}} \right), & i = 1, \ldots, \lambda, \\
\tilde{\Theta}_k + \beta \left( \sqrt[2]{\lambda \theta_{i,k}} \right), & i = \lambda + 1, \ldots, 2\lambda, 
\end{cases} \tag{8}
\]

where $\beta$ is a scaling parameter and $(\cdot)$ stands for the $i$th column of the square root matrix. Sigma vectors go through the nonlinear function $\mathcal{F}$, and the mean and covariance for $\chi_{i,k+1 | k}$ are approximated via the following equations [38], [39],

\[
\chi_{i,k+1 | k} = \mathcal{F}(\zeta_{i,k}), \quad i = 1, \ldots, 2\lambda, \tag{9}
\]

\[
\tilde{\Theta}_{k+1 | k} = \sum_{i=0}^{2\lambda} \eta_i \chi_{i,k+1 | k}, \tag{10}
\]

\[
\tilde{Q}_{\theta_i+1 | k} = \sum_{i=0}^{2\lambda} \eta_i (\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})(\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})^T + Q_Z, \tag{11}
\]

where the transpose operation is shown by $^T$ and the weights are defined as $\eta_0 = 1 - 1/\beta^2$ and $\eta_i = 1/(2\lambda \beta^2)$, for $i = 1, \ldots, 2\lambda$.

\[
\mathcal{Y}_{i,k+1 | k} = \mathcal{H}(\chi_{i,k+1 | k}), \quad i = 1, \ldots, 2\lambda, \tag{12a}
\]

\[
\tilde{Y}_{k+1 | k} = \sum_{i=0}^{2\lambda} \eta_i \mathcal{Y}_{i,k+1 | k}, \tag{12b}
\]

\[
\tilde{Q}_{\theta_i+1 | k} = \sum_{i=0}^{2\lambda} \eta_i (\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})(\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})^T + Q_V, \quad i = 1, \ldots, 2\lambda, \tag{13}
\]

\[
\tilde{Q}_{\theta_i+1 | k} = \sum_{i=0}^{2\lambda} \eta_i (\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})(\chi_{i,k+1 | k} - \tilde{\Theta}_{k+1 | k})^T, \quad i = 1, \ldots, 2\lambda, \tag{14}
\]

Now, the UKF gain $K_G$ is defined as $K_G = \tilde{Q}_{\theta_i+1 | k}^{-1} \tilde{Y}_{k+1 | k}$. The next state estimation and its covariance are applied to the antenna at state $k + 1$ and can be written as,

\[
\tilde{\Theta}_{k+1} = \tilde{\Theta}_{k+1 | k} + K_G(Y_{k+1} - \tilde{Y}_{k+1 | k}), \tag{15a}
\]

\[
Q_{\theta_i+1} = \tilde{Q}_{\theta_i+1 | k} - K_G \tilde{Q}_{\theta_i+1 | k} ^1 T_k, \tag{15b}
\]

while the residual covariance matrix is $Q_{\theta_i+1 | k} = E[v_{k+1 | k} v_{k+1 | k}^T]$.

If the vehicles are still overlapped, the next macro-state will be the statistical interference cancellation; otherwise, the system shifts to the transmit state, as explained in Fig. 2. Table 2 summarizes the main mathematical terms used in the fine estimation and the rest of paper.

3.3 Statistical Interference Cancellation

Let $\mathcal{J}$ be the number of underwater vehicles deployed in the body of water with $\mathcal{J} \leq T_x$, where $T_x$ is the number of buoy’s antennae. Two vehicles $j$ and $i$ ($j, i \in \mathcal{J}$) probabilistically might overlap considering their uncertainty regions, as shown in Fig. 5. If the AODs for each pair of vehicles are $\Gamma _{\theta}^{(j)} = \tilde{\theta}^{(j)}$ and $\Gamma _{\theta}^{(i)} = \tilde{\theta}^{(i)}$, they can be mapped from the $m$th and $(m + 1)$th antennae, $\forall m \in \{1, \ldots, T_x\}$ to their center point, where $\tilde{\theta}^{(j)}$ and $\tilde{\theta}^{(i)}$ are the transferred angles corresponding to vehicles $j$ and $i$ in the $\theta$’s plane. We can conclude that $\tan(\tilde{\theta}^{(j)}) = (1 + d_m / z_m^{(j)}) \tan(\tilde{\theta}^{(i)})$, where $z_m^{(j)}$ is the vertical distance between the depth of vehicle $j$ and antenna $m$, and $d_m$ is the distance between the center point and antenna $m$. As $z_m^{(j)} \gg d_m$, it can be concluded that $\tilde{\theta}^{(j)} \approx \tilde{\theta}^{(i)}$.

As shown in Fig. 5 and given the probability distribution of the vehicles’ positions, interference might occur if the upper boundary in the uncertainty region of vehicle $i$ overpasses the lower boundary in the uncertainty region of vehicle $j$; we call this situation statistical interference as follows.

Statement 6. For random variables $\tilde{\theta}^{(j)}$ and $\tilde{\theta}^{(i)}$, with means of $\tilde{\theta}^{(j)}$ and $\tilde{\theta}^{(i)}$ and standard deviations of $S_{\tilde{\theta}^{(j)}}$ and $S_{\tilde{\theta}^{(i)}}$, we define the probabilistic separability as $\Pr(\tilde{\theta}^{(i)} < \tilde{\theta}^{(j)})$. If the interference occurs, the overlapping area of the distribution functions of $\tilde{\theta}^{(j)}$ and $\tilde{\theta}^{(i)}$, as specified in Fig. 5, represents the statistical interference of two vehicles as $P_{I^{(j,i)}} = 1 - \Pr(\tilde{\theta}^{(i)} < \tilde{\theta}^{(j)})$.

Lemma 1. Interference is modeled by normal probability distribution function with the mean and the standard deviation of $\mu_I = \tilde{\theta}^{(j)} - \tilde{\theta}^{(i)}$ and $\sigma_I = \sqrt{(S_{\tilde{\theta}^{(j)}})^2 + (S_{\tilde{\theta}^{(i)}})^2}$, respectively.
We present a method that minimizes the acoustic beam interference by considering the updated position of each vehicle. This method finds a more focused beam for the overlapping vehicles so as to reduce the interference; at same time, however, this may cause the vehicles to fall out of coverage. There is a trade-off between the probability of interference and the probability of miss; hence, an optimization problem is required to find the beam parameters to minimize the interference while satisfying coverage requirements. By keeping the transmitter antenna’s beamwidth at an optimal value, the proposed method finds a desirable trade-off among three contrasting goals: (i) spreading the signal beam towards the receiver to combat position uncertainty; (ii) focusing such beam to reduce acoustic energy dispersion; and (iii) minimizing interference to other vehicles in the surrounding. In the case that there are more than two interfering vehicles, the interfering vehicles will be sorted from high to low and interference cancellation will be performed for each pair of interfering vehicles, sequentially.

**Lemma 2.** Minimizing the statistical interference is paramount to minimize the ratio of \[ \frac{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)} - \theta'(i)}}{\theta'(j)} \].

**Proof.** Starting from our definition of interference \( P_I^{(i,j)} = P_T(\theta'(i) > \theta'(j)) \), we have,
\[
P_I^{(i,j)} = \int_0^\infty f_1(\theta'(i))d\theta'(i) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_0^\infty \exp\left\{-\frac{1}{2}\left(\frac{\theta'(i) - \mu_1}{\sigma_1}\right)^2\right\} d\theta'(i).
\]

By defining the auxiliary variable \( x = (\theta'(i) - \mu_1)/\sigma_1 \), we obtain,
\[
P_I^{(i,j)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(\mu_1/\sigma_1)} e^{-(x^2/2)} dx = \Phi\left(\frac{\mu_1}{\sigma_1}\right),
\]
where \( \Phi(.) \) is the Cumulative Distribution Function (CDF) of the standard normal distribution. Therefore,
\[
P_I^{(i,j)} = \Phi\left(-\frac{\theta'(j) - \theta'(i)}{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)}}\right) = 1 - \Phi\left(\frac{\theta'(j) - \theta'(i)}{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)}}\right),
\]
where the equality is concluded via the rotational symmetry characteristic of function \( \Phi(.) \). From (19), it is inferred that
\[
\frac{\theta'(j) - \theta'(i)}{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)}} = \Phi^{-1}(1 - P_I^{(i,j)}).
\]

Here, \( \Phi^{-1}(.) \) is the Probit function and \( \Phi^{-1}(1 - P_I^{(i,j)}) \to \infty \) when \( 1 - P_I^{(i,j)} \to 1 \). Therefore, the left side of (20) approaches \( \infty \) when \( P_I^{(i,j)} \to 0 \). It proves that we can minimize the statistical interference by minimizing \[ \frac{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)}}}{\theta'(j) - \theta'(i)}. \]

Let \( \Theta' = (\bar{\theta}'(j), S_{\theta'(j)}, \bar{\theta}'(i), S_{\theta'(i)}) \) be the vector of variables. An optimization problem with the objective function introduced in Lemma 2 is cast as follows.

**Given:** \( W_i^{(j)}, \Gamma_i^{(j)}, S_{\theta'(j)}, \bar{\theta}'(i), \theta_0(j), \rho, N, \alpha, J = i, j; \)

**Find:** \( (\bar{\theta}'(j)^*, (S_{\theta'(j)}^*)^* \min\limits_{\theta'} \Lambda(\Theta') = \frac{\sqrt{(S_{\theta'(j)}^2 + (S_{\theta'(i)}^2)}}}{\theta'(j) - \theta'(i)} \)

\[ (21a) \]
\[ \text{s.t.} \quad 2(\bar{T}_{N-1, \alpha/2} S_{\theta'(j)}^2 N^{-1} \leq W_i^{(j)}, J = i, j, \]
\[ (21b) \]
\[ \bar{\theta}'(j) + (\bar{T}_{N-1, \alpha/2} S_{\theta'(j)}^2 N^{-1} \leq \theta_0(j), \]
\[ (21c) \]
\[ \bar{\theta}'(i) - (\bar{T}_{N-1, \alpha/2} S_{\theta'(i)}^2 N^{-1} \geq \theta_0(i), \]
\[ (21d) \]
\[ \bar{\theta}'(j) + (\bar{T}_{N-1, \alpha/2} S_{\theta'(j)}^2 N^{-1} \geq \theta_0(j) + \rho S_{\theta'(j)}^2 4, \]
\[ (21e) \]
\[ \bar{\theta}'(i) - (\bar{T}_{N-1, \alpha/2} S_{\theta'(i)}^2 N^{-1} \leq \theta_0(i) - \rho S_{\theta'(j)}^2 4. \]
\[ (21f) \]

This problem is a constrained multivariable fractional nonlinear programming as a ratio of chosen real-valued functions from a set \( D \) of space \( \mathbb{R}^4 \). Both the numerator and denominator are positive for all values of \( \Theta' \) which are defined by the constraints domain set \( D \). The numerator is a norm function which is convex [40], while maximizing the denominator minimizes the objective function. Since \( \bar{\theta}'(j) > \bar{\theta}'(i) \), the maximum value of the denominator would be \( \max(\bar{\theta}'(j)) - \min(\bar{\theta}'(i)) \) over the domain. However, the constraints optimize the mentioned value to satisfy the interference-miss trade-off.

The objective function in (21a) minimizes the interference area by maximizing the distance between AODs of overlapping vehicles while their HPBWs in the numerator are minimized. Moreover, (21b) controls the beamwidth such that it does not exceed the initial estimated beamwidth. While constraints (21c)-(21d) keep the new beam boundaries, i.e., \( \theta_0^j \) and \( \theta_0^i \), inside the uncertainty regions, we prevent the beams to become too narrow.
is achieved for a specific threshold through the following steps.

Step 0—Initialization: Find the initial value for $\psi$.

Step 1—Solve (22) and find $\Theta^{(i)}$; if the value of $\mathcal{L}(\psi)$ does not satisfy the threshold, move to the next step.

Step 2—$\psi = \Lambda(\Theta^{(i)})$.

Step 3—Solve (22) and update $\Theta^{(i)}$ with the new result; if $\mathcal{L}(\psi)$ fulfills the threshold, stop; otherwise, repeat steps 2 and 3 until the threshold is satisfied. The optimum value is the one which is associated with the most recent $\Theta^{(i)}$.

The solution of the presented program determines the optimum values of angles for minimum probability of interference at a tolerable miss rate. We define the probabilistic retransmission rate as a measure of probabilistic failure or disconnectivity, where the former is the result of interference and the latter is the consequence of missing the vehicle. This metric is calculated as in (23) if it is the result of interference, whereas it is computed using (24) in case of missing the coverage.

\[
\mathcal{R}_f^{(j)}(\rho) = \frac{P_f^{(i,j)}}{\int_{\Theta_L^{(i)}}^{\Theta_U^{(i)}} f_{\psi^{(j)}}(\theta^{(j)}) \, d\theta^{(j)}}, \quad (23)
\]

\[
\mathcal{R}_m^{(j)}(\rho) = \frac{\int_{\Theta_L^{(i)}}^{\Theta_U^{(i)}} f_{\theta^{(j)}}(\theta^{(j)}) \, d\theta^{(j)}}{\int_{\Theta_L^{(i)}}^{\Theta_U^{(i)}} f_{\psi^{(j)}}(\theta^{(j)}) \, d\theta^{(j)}}.
\]  

In both cases, the buoy retransmits the same data packets after re-tuning the transmit parameters.

Fig. 6 and Fig. 7 show the sequence of events that occur in multiple transmission rounds for a downlink/uplink communication between the buoy, as the initiator, and vehicles $i$ and $j$, when vehicles are inside the buoy’s transmission range. We assume an asymmetric and separate spectrum utilization for downlink and uplink channels; i.e., a larger bandwidth in downlink for data transmission and a narrow-bandwidth uplink channel for acknowledgments and new vehicle’s locations. Downlink exploits the proposed SDMA, while for the uplink, a polling technique, controlled by the buoy, is used to determine which vehicle is eligible to use this feedback channel at a given time. Each vehicle responds to the polling packet and sends back its new position information to the buoy, which is used in the next round of MAC decision process and spatial parameters calculations.

Fig. 6. Timeline showing the interaction between the buoy and vehicles $i$ and $j$ in the transmission range. Comparison between different incidents is shown in different transmission rounds. In transmission round (I), reliable transmission is ensured since the vehicles are spatially separate; however, in rounds (II) and (III), the interference occurs.
vehicles send back their NACKs in response to buoy’s polling, one at a time. If the vehicle is out of coverage, then the data will be missed and other actions are required after a timeout. This incident is shown in transmission round (V). Fig. 8 sketches the possible situations after the data is missed. In round (VI), tuning the spatial parameters solves the coverage problem, so the ACK signal is received. In round (VII), this tuning causes an interference between the vehicles, so the retransmission fails. Buoy is notified of missing the retransmission data by a timeout signal in round (VIII). It is concluded that MAC has not accomplished a successful retransmission, so it switches to hybrid SDMA macro-state, which is discussed in the following section. Note that the proposed method makes two assumptions: (i) for short/medium transmission range, in which the vehicle is within the transmission range \(D\) of buoy, the above discussion works well; (ii) for farther distances, in the zone \((D - R, D + R)\) and above, data may be missed and timeouts occur due to vehicles going out of transmission range. This scenario is covered in the algorithm.

### 3.4 Spatially Non-separable Probabilistic SDMA

In this section, we discuss a solution for the situation in which the vehicles are fully overlapped and the separation is not possible in space. Vehicles at the same azimuth and elevation angles but different distances from buoy, are categorized as non-separable vehicles. We propose a hybrid TDMA-SDMA method, called T-SDMA, that uses time as the second domain and leads to an interference-free MAC solution for time-insensitive applications. Synchronization between the vehicles is not required since the solution is presented for downlink—buoy to vehicle—transmission. All data exchanges will be made through the buoy as the primary controller of the links. However, we assume a separate feedback channel for the uplink; therefore, when two or more vehicles are in the same area a polling packet is exchanged between the buoy and each vehicle separately to avoid any collision. Corresponding vehicle will be allowed to transmit through the feedback channel.

Firstly, clusters of non-separable vehicles are formed; then time sharing is applied inside each cluster. To compare this method with conventional TDMA, we define the time slot usage ratio as,

\[
\tilde{R}_{TS} = \frac{n_c}{M_2 \cdot \max(M_1, M_3)}
\]

where \(n_c\) is the total number of time slots dedicated to each vehicle of cluster \(c \in \tilde{c}\) in every \(M_2\) frame. If \(|\tilde{c}| > T_k\), an additional external clustering is required, which leads to an inter-cluster time sharing defined by parameter \(M_2\). The denominator of (25) denotes the total number of time slots; \(M_1\) is the maximum number of vehicles in the clusters that are communicating with separated antennae, while \(M_3\) is the maximum number of vehicles per cluster in the group of inter-clusters. By selecting \(\max(M_1, M_3)\) as the number of time slots of each frame, we guaranty that at least one time slot is dedicated to every vehicle within a cluster. The time slot usage ratio for conventional TDMA is calculated as \(R_T = 1/M_T\), where \(M_T\) is the total number of time slots, i.e., it is equivalent to the number of vehicles. We define rate efficiency of the proposed T-SDMA technique over the conventional TDMA as \(\eta = \tilde{R}_{TS}/R_T\).

The effective data rate per user over the total transmission time is defined in [43] as \(1/NB \log_2(1 + SNR)\), where \(N\) is the number of time slots, \(B\) is the channel bandwidth, and \(SNR\) is the received Signal-to-Noise Ratio. We find the effective data rate of each vehicle in one transmitting frame, for T-SDMA as,

\[
\tilde{R}_{TS} = \frac{N_p L_p n_c/M_2}{\max(M_1, M_3) T_s} = \frac{1}{\max(M_1, M_3)} B \log_2(1 + SNR_{TS})
\]

where \(N_p\) and \(L_p\) are the number and length of packets, respectively, and \(T_s\) is the time duration of each time slot. The effective data rate of each vehicle in one transmitting frame for the conventional TDMA follows \(R_T = (N_p L_p)/(T M_T) = 1/M_T B \log_2(1 + SNR T)\). Therefore, the ratio of the effective

![Fig. 8. Possible situations that might happen when the data is missed, i.e., the vehicle is out of the buoy’s antenna coverage but within the buoy’s transmission range.](image-url)
We consider a short/medium range transmission (less than 2 km) with an anchored buoy and several moving vehicles. Assume each vehicle can communicate with one of the buoy’s directional hydrophone arrays. An underwater acoustic channel is simulated with a limited number of separable paths in which the first path is the strongest one (lowest transmission loss) and with the smallest delay. The required specifications of the underwater acoustic channel for the simulation are extracted from the Kauai Acoms MURI (KAM08) experiment at the western coast of Kauai, HI, USA [44]. The normalized delay profile and the phase response of the sample emulated channel is depicted in Fig. 9, where an anchored vertical hydrophone array with 3.75 m inter-element spacing at the depth of 96 m communicates with a source towed by a surface ship.

The other simulation parameters are listed as follows. The channel bandwidth is 5 – 25 kHz, the sampling rate is 50 kHz, and an underwater ambient noise leads to an $SNR \in [0, 20]$ dB. We use the Gaussian noise model, introduced in [26], in which the overall power spectral density of the ambient noise is assumed as $N(f) = N_r(f) + N_s(f) + N_w(f) + N_{th}(f)$. Here, $N_r, N_s, N_w$, and $N_{th}$ stand for the turbulence noise, shipping activity, wind-driven noise, and thermal noise, respectively. The vehicles are randomly deployed at different depths from the surface [0, 500] m and horizontal distances [100, 2000] m. After converting these initial locations to the required antenna parameters at the buoy, Table 3 reports the initial state via an initial interval estimation. Vehicles move with a constant speed of 0.5 m/s and transmit a new position information when the buoy polls them, according to the explained procedure in the algorithm, every 10 s for the sampling interval of 50 minutes. Finally, we set the percentage of confidence, i.e., $(1 - \alpha)$ in (1), to the common value of 95%.

Algorithm 1 reports the set of rules in pseudo-code under different circumstances. The new location sample is observed, the interval estimation is updated given the probabilities of possible cases, i.e., keeping the current AOD and beamwidth if the vehicles are far apart, updating the AOD and beamwidth by sliding the estimation area of the samples, or switching to a new uncertainty region by accumulating the samples. It will be applied to a Kalman estimator to decide on the next upcoming state. While the neighboring vehicles overlap, the optimization problem is solved. If it returns the optimized values, the vehicles are assumed separable and a similar procedure as described for separate vehicles will be applied to them; however, MAC will decide on the interference-miss trade-off. Since it is a probabilistic MAC, transmission is performed by taking the risk of miss. If the ACK message gets back, then it will continue the transmission without any interruption; otherwise, retransmission will be performed in the next round. Parameter $SW$ is used to count the attempts in order to get the optimal values. If it fails, then it triggers the algorithm to apply the hybrid MAC solution, i.e., T-SDMA, to cover the problem of non-separable vehicles.

### 4 Performance Evaluation

We consider a short/medium range transmission (less than 2 km) with an anchored buoy and several moving vehicles. Assume each vehicle can communicate with one of the buoy’s directional hydrophone arrays. An underwater acoustic channel is simulated with a limited number of separable paths in which the first path is the strongest one (lowest transmission loss) and with the smallest delay. The required specifications of the underwater acoustic channel for the simulation are extracted from the Kauai Acoms MURI (KAM08) experiment at the western coast of Kauai, HI, USA [44]. The normalized delay profile and the phase response of the sample emulated channel is depicted in Fig. 9, where an anchored vertical hydrophone array with 3.75 m inter-element spacing at the depth of 96 m communicates with a source towed by a surface ship.

The other simulation parameters are listed as follows. The channel bandwidth is 5 – 25 kHz, the sampling rate is 50 kHz, and an underwater ambient noise leads to an $SNR \in [0, 20]$ dB. We use the Gaussian noise model, introduced in [26], in which the overall power spectral density of the ambient noise is assumed as $N(f) = N_r(f) + N_s(f) + N_w(f) + N_{th}(f)$. Here, $N_r, N_s, N_w$, and $N_{th}$ stand for the turbulence noise, shipping activity, wind-driven noise, and thermal noise, respectively. The vehicles are randomly deployed at different depths from the surface [0, 500] m and horizontal distances [100, 2000] m. After converting these initial locations to the required antenna parameters at the buoy, Table 3 reports the initial state via an initial interval estimation. Vehicles move with a constant speed of 0.5 m/s and transmit a new position information when the buoy polls them, according to the explained procedure in the algorithm, every 10 s for the sampling interval of 50 minutes. Finally, we set the percentage of confidence, i.e., $(1 - \alpha)$ in (1), to the common value of 95%.

We explore a scenario where two vehicles start their mission from different locations with a common target; therefore, their trajectories tend to converge to the same area. Fig. 10 shows the simulation assumption on the trajectory of these vehicles which is used in performing the system simulation. However, there are uncertainties in the estimation of the location. To handle these uncertainties, we adopt the statistical approach to estimate the position of each vehicle. Based on the procedure, explained in Fig. 4, MAC decides on the next estimate of each vehicle’s AOD and beamwidth. In Fig. 11(a), the resultant AODs and beam boundaries are plotted for two vehicles versus time. The figure shows that after 15 minutes the corresponding beams penetrate each other, and hence interference occurs.

Figures 11(b), (c), and 12(a) evaluate the corresponding parameters after the interference occurs. In particular, Fig. 11(b) shows the probability of interference as time passes and vehicles get closer to each other; by updating the antenna beam direction and beamwidth via the optimization problem, the probability of interference decreases. However, this outcome depends on the value of $\rho$, which is defined by MAC to control the optimized beamwidth.
an appropriate measure for MAC to choose a proper value for reflect the effect of both interference and miss probabilities, it is for almost As an example, $\rho$ by increasing the decrease in the corresponding probability of miss by time and additional inter-clustering.

Fig. 13. Clustering scenario in T-SDMA including 5 clusters and an

Based on the evaluation of retransmission rate, when its and to regulate the interference-miss trade-off. Fig. 11(c) describes the decrease in the corresponding probability of miss by time and by increasing $\rho$, which contradicts the interference trend. When the objective function of the optimization problem does not change in a feasible direction, the solver finds a local minimum that satisfies the constrains, which may lead to a sudden change in the curves. In Fig. 12(a), the variation of retransmission rate is studied as time passes and for different values of $\rho$. Since this parameter reflects the effect of both interference and miss probabilities, it is an appropriate measure for MAC to choose a proper value for $\rho$. As an example, $\rho = 0.5$ keeps the retransmission rate below 20% for almost $2/3$ of the transmission window, i.e., 20 min.

Based on the evaluation of retransmission rate, when its projected value surpasses a tolerable quantity, MAC switches to T-SDMA mode and clusters the vehicles. As an example, in Fig. 12(a), when retransmission rate crosses 50%, it seems more reliable to switch to T-SDMA. In order to provide an example of the proposed hybrid T-SDMA, we investigate the scenario that is displayed in Fig. 13. We assume 11 vehicles are categorized in 5 clusters, which communicate with 4 antennae at the buoy. MAC performs an additional inter-cluster time sharing for the last two clusters. Figures 12(b) and 12(c) compare the performance of the proposed hybrid T-SDMA method with the conventional TDMA. Fig. 12(b) shows the time slot usage ratio and the rate efficiency and confirms that T-SDMA outperforms the conventional TDMA. Fig. 12(c) compares these methods in terms of SINR and effective data rate per vehicle. Appropriate clustering along with the beam-forming ensures we achieve a gain for maximum achievable data rate in T-SDMA compared to the traditional TDMA, while there is no interference between the vehicles. In other words, we exploit space to increase the rate compared to the traditional TDMA, which is directly related to the maximum required SINR.

In Fig. 14(a), the probability of interference of the proposed probabilistic SDMA is compared with the non-probabilistic SDMA. In the latter, we assume that the transmitter estimates the probability of interference of probabilistic SDMA is less than the deterministic method, since our method updates the beam specifications by statistical calculations. This superiority increases when probabilistic SDMA applies the optimization to it.
Spatially separable and non-separable scenarios were studied and the position of the vehicle and focus the beam, respectively. A two-stage estimation technique was presented based on interval leverage inherent position uncertainty of the moving vehicles. A novel probabilistic MAC based on Space Division Multiple Access (SDMA) for short/medium distances was proposed to achieve reliable communications and interference mitigation for a team of AUVs in the underwater environment. In this paper, an efficient MAC protocol is a necessity for a team of AUVs in the surrounding. As each vehicle moves and more samples are accumulated to make a better focused beam, the antenna gain and the directivity change. These variations lead to a change in SINR which is reflected in the data rate variation. In Fig. 14(c), the validation of UKF state estimation is verified by plotting the residual signals. The corresponding curves show that the residual magnitudes are small, their mean values are zero, and the autocorrelation functions are zero except at zero lag. Finally, in Fig. 15, we provide the beampattern of one of the buoy’s arrays while it is steered towards a vehicle at sampling elevation and azimuth angles of 25° and 80°, respectively. The figure shows the 3D pattern, elevation, and azimuth cut, when the frequency equals to the maximum value of 45 kHz.

5 CONCLUSION
Achieving reliable communications and interference mitigation for an efficient MAC protocol is a necessity for a team of AUVs in the time- and space-varying underwater environment. In this paper, a novel probabilistic MAC based on Space Division Multiple Access (SDMA) for short/medium distances was proposed to leverage inherent position uncertainty of the moving vehicles. A two-stage estimation technique was presented based on interval estimation and Unscented Kalman Filtering (UKF) to estimate the position of the vehicle and focus the beam, respectively. Spatially separable and non-separable scenarios were studied and an optimization problem was solved to minimize the statistical interference. The method was extended to the scenario of non-separable vehicles via a hybrid T-SDMA solution. Simulation results demonstrated that the proposed approach could handle the interference, while the vehicles were moving, and so it could achieve a high data rate and reliability. Note that the proposed method may be used in the uplink too if there is the possibility of mounting a beamformer and an array of hydrophones at each vehicle; also, since currently the coordination is performed at the buoy in a centralized fashion, a distributed in-network coordination among the vehicles in the uplink would be needed.

REFERENCES


